Preparation for fractals talk

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You may know how to solve equations like this:

$$x^5 = 20$$

The solution is written as $\sqrt[5]{20}$, and there is a button for typing this in to your calculator. (On some calculators, the button looks something like this: $\sqrt[5]{\square}$.)

But what about an equation like this:

$$5^x = 20$$

There is a button for calculating powers such as $5^{1.6}$ on your calculator (it may look something like this: \square^{\square}), so one way we could find the value of x is by using trial and improvement; here is one possible sequence of trials:

x	5^x	low or high
1	$5.00\ldots$	low
2	25.00	high
1.6	13.13	low
1.7	15.42	low
1.8	18.11	low
1.9	21.28	high
1.85	19.63	low
1.86	19.95	low
1.87	20.28	high
1.863	20.05	high
1.862	20.02	high
1.861	19.98	low

and so x lies between 1.861 and 1.862.

Now solve the following equations in a similar way: $3^x = 2$ and $4^x = 15$.

Bonus activity

There is a button on your calculator which you can use to solve these equations directly. Can you find it? You will know that you are right if you can put 5 and 20 in and get 1.861... as an answer, and similarly for the other two equations that you have solved by trial and improvement.

And finally...

Please bring a calculator to the session; you will need it to solve equations like these!