Mathematics session

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SUMMARY KEYWORDS

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04:06

Good afternoon everybody, and we've got lots of us here today again which is fantastic so just bear with us while we wait for people to come in from the waiting room, it just takes a second. Okay, excellent. I think that's everybody here today. So hello everybody, we already met on Monday filler we do see you all here again today. So just to remind you I'm Caitlin and I'm currently the Outreach Officer here at Lucy Cavendish college and my math knowledge is absolutely terrible so I will not be leading the session and I'll be letting my colleague student and Claire do that and I'll let them introduce themselves in a second. And other than that, and once I've started the session off I'm going to be disappearing and letting everybody get on with it so I hope you have an excellent session, as with last time. We're using some closed captions which are auto generated so everybody can follow along. So if you're wondering what's happening at the bottom of your screen, it's just the live captions. So I'll let the session get started, I think we're ready to go.

05:11

Thank you. Hello and welcome, and so I'm Julian Gilby, and I'm going to be leading the session and supporting me today is Claire Metcalf. So Claire works for enrich and runs the step support program. Don't say hello.

05:27

Hello everyone. So I'll also be on the chat so if you've got any questions, what have you put them in the chat and I'll be responding to them.

05:40

And so I'd like to tell you a little bit about my life as a mathematician and what I've been up to give you some sense of where this session fits in, what I've done and the sorts of things you might end up going on to do, if you do choose to study math. So, are you studying maths at school, like you are, and I went on started studying maths at university. And after I studied maths at university I really liked the very theoretical side of maths. And so I stayed on study for a PhD I became a doctor of mathematics. I do operations on numbers. And after I'd done my research in in very theoretical maths, I was thinking, not quite sure this is the right thing for me. What should, what could I do instead. And I looked around, I ended up going into school teaching, so I taught in schools for about 10 years at secondary school GCSE a level and so forth, and had a fantastic time. And after about 10 years of doing that I got involved in a project called underground mathematics and then I did some work with enrich you might have come across enrich and underground mathematics you may come across when you start your a level courses next year it's a collection of wonderful wonderful teaching resources for a lot of mathematics. And after that, the funding finished for that, although there's potentially some new funding to continue a project, but at the time the funding finished. What do I do now. I spoke to lots of people they said Julian you love maths, why don't you go into maths thought, actually that's a really good idea. And everyone's talking about this newfangled area called data science. So I thought, maybe I should go do some data science. So I talked to some people about getting into data science, and ended up starting a project in somethings of data science II in the math department. And so now I'm doing maths and data science, and working with all sorts of you doing really cool stuff, so I thought I'd show you the project I'm working on just for a couple of minutes before we start the main topic of today. So I'm now working with doctors, medical doctors, and in particular pathologists to try and help pathologists, diagnose celiac disease. Okay, so what is celiac disease. It's a condition which affects maybe 1% of the population. And it's a problem in the small intestine the bit that comes just after your stomach through your digestive system so food goes in your mouth down through your esophagus into your stomach comes out of the stomach into the small intestine and Celiac disease is an autoimmune condition which is triggered by eating gluten, and if somebody was celiac disease eats gluten so that's food that you find him bread in wheat, for example, it can really inflame the small intestine. And if the small intestine is inflamed, that can be all sorts of problems stomach problems, pain, trapped wind really uncomfortable pleasant, very nonspecific conditions, but it can lead to these conditions, how do we diagnose celiac disease. Well what we do is we stick a tube or Dr will stick a tube down the person's throat through their stomach into the small intestine, take a small bit of tissue from the small intestine. Give it to pathologist who then sticks it under a microscope and looks at it. Let me show you an example of what that looks like. If I can share screen. Here's an example. And on the left this is what a person with a normal, healthy, what's called dudina and that's technical name for this part of the small intestine, what it looks like under a microscope very very fine details, and under the microscope, you see that the gutters line this small intestines line with these little finger like protrusions and that's where your food gets absorbed where the, the nutrients and food are absorbed into the body and this part of the intestine. But in celiac disease, all of those, this is somebody very severe celiac disease. All of these lovely finger like protrusions have all but disappeared. So the question is can we use mathematics and artificial intelligence and things like that to try to look at these slides, and without needing a human to be involved, to say, this is normal. This is celiac disease. So that's what I've been working on. And there's also statistics involved in it, so I looked at the data we had from the, from the patients. Let me get this are

10:32

from current slide. So, we looked at some of the clinical data, some of the information about people's ages and the celiac disease. So celiac disease, About 1% of the population have celiac disease. And we looked at our sample 465 biopsies to form 65 different patients. And we said how many of them are normal 215 How many of celiac 250. But then we looked at different age groups, so those patients who were under the age of 55 only 19% were normal 81% were celiac, but in patients over the age of 55. It was completely the opposite way around 70% of them were normal 30% were celiac. That's what the figures tell us, and then this mathematicians, we have to interrogate and think and talk to doctors and talk to other people and think, What is going on here we could come with all sorts of explanations for this. But what could some possible reasons be for why we get to such a difference in the different age groups. So I'm going to give you a minute to think about that question. And I'm going to give you a minute to think about that question and then I'll ask you to type your suggestions into the chat, and then we can see what some various different suggestions are for why this might be the case. So, have a minute to think about it and then we'll look at the chat. Sorry.

12:33
So another 30 seconds before you type suggestions about five seconds. Okay and if you can share suggestions with everyone please so 321 Go.

13:17
So if you could share it, send it to everyone please rather than just to me then everyone can see the suggestions.

13:32
We're getting some lovely suggestions. So I some of them was sent directly to me but next time please if you could send them to everyone that would really help and then we can see what everyone's thinking, as Mike maskel So we've got suggestion maybe genetics is something to do with it. Well, that's an interesting idea. Younger people tend to have better immune systems so they're less likely to develop celiac disease. Oh, that's an interesting idea, except the young people seem to be the ones who are getting celiac disease so it seems to be the wrong way around but maybe there's something to do with the immune system, maybe because Celiac disease is an immune condition younger people have better immune systems and that's why they're more active and they're more likely to show signs, whereas older people have a weaker immune system so they don't have those same problems, so you could turn it the other way around. That could work that could be a reason. And the old friends hypothesis, I'm afraid I don't quite know what that is. Different epigenetics all that's interesting. I don't know any of these could be factors, I'm not saying any of them are right or not, these are all possible differences immune systems yes the older you get, the less efficient your organs become all that's, that's, again, that same idea. Older people are more likely to have other reasons for the same symptoms wall. That's a really interesting one, if the older people could have lots of reasons for having stomach symptoms or digestive systems, and we've biopsy lots of them, then the older people are more likely to have lots of other reasons for not being celiac for other than things celiac so that's a really interesting idea, young people is more common. Yeah, it could be more common in younger people, slow rate and digestion and older people. These are fabulous suggestions. Good job guys. crossfade celiac in the imbalance sample taken perhaps. That's a really interesting question imbalanced sample. We have no idea where these who chose these particular biopsies so yes that is one of my big concerns is when don't have a representative sample, and so that's why, why the figures are, are, as weird as they are really good idea. older people have low metabolisms again single idea, depends on diet. Yes, so celiac disease only shows itself if you're eating gluten. So maybe but I'm possible. Celiac is common in younger age groups because they have better immune systems which read more strongly. These are wonderful. I'm not going to be able to read them all because that goes off, it's like 20 of them their day is amazing. Old friends, your body gets used to it. Ah, so maybe that's an idea that you're, it's shows less when you're older because, because your system gets used to. Okay, so we've got lots of explanations, and now I as mathematician, can't decide between them. I now need to work and collaborate with my colleagues in the medical side to say here are 2030 solutions or

suggestions for why we're seeing this, and unless we know which one of them is, or which ones, it might be more than one which ones of them are most likely, I don't know how to interpret this data. And if I don't know how to interpret the data, I don't know how to use that sort of data to make future predictions, because I get in a new biopsy and I want my computer system to take all the data, including the agent Tell me, how likely is this person to have celiac disease. And so, I need to take into account because if I take the wrong reason into account, then my predictions are not going to be good enough. So that's the sort of thing I do my day job, I hope you found that interesting. What I'll do the session finishes at 330 but after 330 I'm happy to stay for five minutes, so if people want to ask questions about this or any other things that I myself or Claire, have done in mathematics over the years are happy stay for a few minutes at the end and you can ask those questions. So thank you for that. So let me let me return to, to what I'm planning to talk about today, which is the main, the main subject is, I'm going to talk about fractals. So, let me, let me introduce this briefly when I was an undergraduate I did a summer project studying fractals and since then I've dabbled in fractals and they come up all over the world, all over the place in math. And, and so I thought I'd share some of the beauty of this, this area of maths with you. It's something that's not taught at school and it may be touched upon in university. It's a fairly specialized subject but at more advanced levels it comes in all over the place. So, what is a fractal Well,

18:28
let's go back in time. We study geometry at school, you're very familiar with geometry and the geometry we studied at school was essentially written down and formalized by this this gentleman. Gentleman by name of Euclid, this is not what he looks like. This is a picture drawn in probably the medieval period as a front piece to a book. So maybe, maybe, or maybe it's even later than that maybe it's 16 1700 I'm not sure exactly when this photos from this picture is from, so would can't. You could lift about two and a half 1000 years ago, nobody knows what he looked like he wrote a very famous book, which is called in English translation, the elements, consists of 13 books, serve as a collection of books. The elements, and in it he describes geometry and arithmetic and basically lots of things we we know about so you've studied the highest common factors and lowest common denominators, you've studied areas of shapes you've studied triangles you've studied angles, all these things you've studied, and you're studying mathematics that was already known about two and a half 1000 years ago, I think mathematics is an amazing subject, it's the only subject I'm aware of where we still in school today teach the ideas that were current two and a half 1000 years ago, most mathematics we teach you. Up to GCSE was either Euclidean maths of two and a half 1000 years ago, or is a bit later so the introduction of algebra which in some forms was already about 2000 years ago but in its current form using x's and y's and Zed is maybe 400 years old, but a lot of maths we teach us, ancient no other subjects in school will you learn things. Almost without exception, will you learn things that were current two and a half 1000 years ago, Unless you study class, classical breeding. Okay, so you could study shapes, and this going to be quick fire round I'm going to put up a shape and can you type the name of the shape into the chat. So let me find. So what's this shape. It's a circle fantastic, and this one. And you're all spot on, it's a square. And what about this one. Oh you are on fire today, This is brilliant. Okay, what about this shape.

21:04
I've seen lots of trees or trees or question marks or confusion, irregular. Okay, suddenly, Euclid fails. It's a mix of shapes, Euclid fails us, Euclid hasn't described the shape of a tree, those who are trees a

shape. I mean trees and objects, maybe I mean, would you, I mean if I asked you to draw a tree would you draw a tree that looked like that or would you maybe draw a tree that looks like one of those. And, and we realize that actually language for describing shapes is remarkably limited. And during today's session I'm going to talk about some of the things that make trees tree like we won't get as far as coming up with accurate descriptions or good descriptions, these trees that will take us a little bit further than what we'll have time for today, but it's going to give you some sense of the structures of trees are actually really interesting. And the mathematics of trying to understand the structure and the shape of trees, is very much 20th century mathematics was only invented in the early, early 1900s early to mid 1900s But now it's used if you go and watch an animated movie, like a Pixar movie and they have trees in them, trees which are computer generated, how to generate realistic looking trees for computer animations. And the answer is using the sorts of maths, we're going to start looking at today. So the non Euclidean shapes phrase we could call it a fractal. So some people suggested could it be a fractal yes we might we'll describe it as a fractal. And we will see what that actually means over the course of the next hour or so. Here are some other shapes which have that sort of very detailed structure of top left we've got an example of Fern, bottom left we've got a snowflake, again very different but you've got these sort of very, very fine details. And on the right one of my favorites list, veggie favorite it is not a good word, but never mind. One of my favorite vegetables for sheer beauty is called a romanesco cauliflower, and you can see it's performed badly spirals and spirals themselves to form the small spirals and if you're really really lucky you might find one in the supermarket every so often but quite rare. And yes, they repeat the same pattern again, smaller and smaller and smaller thank you well. And then maybe Fibonacci sequence in there as well. Absolutely, there's a, there's a whole area of mathematical biology which studies the question of shapes in, in plants and why do you get these spirals forming and the Fibonacci sequence appearing them so some very nice comments that. So, let's talk about now, what fractals are, and I'm assuming many of you will not have come across fractals for so this is what we're going to explore today, what they are, how do we describe them. Where do they occur and I'll try and share some more mathematical examples of fractals with you. We won't be able to cover everything in, in the PowerPoint but I've put in a variety of things so we'll see how time goes. I also will entirely failed to tell you the answers to any of these, because they're really don't have good answers. I can give you a feeling of the sorts of things that might be fractals and give you some suggestions how we might describe them. And some examples of where they occur, but I won't be able to do better than that. Primarily because there aren't very good answers. Okay so first question what are they so early in the 20th century, as I said, mathematicians are starting to explore geometry in a very, very much more formal way than they had before. And so all sorts of various different directions, and the direction that I'm going to look at with you today is a direction was started with people like, And I'm not going to be able to pronounce his name I'm sorry, that's five Shere Pinsky, who's a Polish mathematician, and he was a logician, he thought very hard about the foundations of mathematics and logic and he was thinking about sets of points and came up with a shape which bears his name today, which is quite strange. So let's explore his triangle. So what he did was he started with a triangle. I can cope with this this is a nice equilateral triangle.

25:47

And he said, if I take my triangle, and I take a smaller triangle half the size turned upside down, I can cut it out of the big triangle. I'm sorry that the corners don't exactly line up with the outside line this as best as I can do on PowerPoint so you just have to imagine it's exactly from, from this point to this point to this point. So take out that smaller triangle. So I've now got three triangles three blue triangles, and I take out a small triangle for each of them. So now I've got more triangles. And then from each of those I take out another triangle. And I could keep on going. And I'll take up another triangle and this is as far as I had patience to do in PowerPoint, but you could imagine in your mind what would happen I could go and take out more than take out triangles from the rest and then check out triangles again and again and again and again and go on for ever doing that to take out triangles one after another after that. So layer upon layer. That's a interesting little shape. But since he wasn't satisfied with drawing pretty pictures. He wanted to ask mathematical questions about these, these pretty mathematical shapes. So, a simple question you could ask is what's its area. Okay so let's, let's start and for this I'm again going to use the chat. So what I'll do is I'll ask the question, and then I'll give you a certain amount of time, and then I'll say, ready 321 Go and you can type the answer into the chat, or you can type the answer but don't hit enter, until I say go, and if you can share it with everyone then we can all see Do we all agree on the answers or do we do, we have some sort of confusion or disagreement. So let's start with the initial triangle, and ask what its areas. And I don't know what the units are so let's just say for the sake of argument that this big blue triangle. This is level zero or stage zero. This has area one are now going to take out this smaller equal lateral triangle which goes from the midpoints of each of the sides. What's the remaining blue area. I started with area one please don't type in yet, please don't type in yet. Okay, wait. Okay, we, we actually failed on that one. Let's try it again. Okay, so several people have typed in the answer. It is indeed three quarters, because I've got three. Next time, Please wait until I say go on stood. Wait, have patience. Okay, so I took out one small once one smaller triangle and I'm left with three quarters of the original shape. Okay, I'm now going to take out middle triangle for each of those, what's the remaining blue area. Wait, I'll give you 20 seconds think about it and then you can go

29:15
321 Go, well hey, loads of answers. Oh, I've got lots of nine sixteenths, I've got a few three quarters squared. And I've got one decimal naught point 5265 I'm not going to manage decimals because it's going to get very, very long numbers very quickly. But nine sixteenths or three quarters squared I think we have agreement on that one. So guess what the next question is going to be what happens on next one I'll give you 20 seconds

30:11
321 Go. Okay, we've got lots of mostly agreed. Okay, we've got some argument between whether it's 2706 for these numbers getting too big, think of that is three quarters cubed or 81 over 256, which is three quarters to the four. Is it three quarters cubed or three quarters to the four. I had at this point. Oops, I had three quarters squared, and I'm taking for each of the blue ones, I'm taking three quarters of it. When I take out that. So is three quarters of three quarters squared. So, as I think majority thought three quarters cube. This is the only point at which decimals might be useful. And what happens if I go on forever and I just keep going and keep going and keep going. So I can only sort of demonstrate a tiny bit more but let's say I keep on going forever, what's the final error going to be when I've gone on forever. Oh, I should have I should have said Please wait, my apologies. I've got zeros or Infinites. Is it zero or infinity, or, or, actually. Okay, so if you have a calculator to the ready. What you might want to do is calculate what's three quarters, what's three quarters squared what's three quarters cubed, what, three quarters four what's happening to numbers so I'll give you 20 seconds 30 seconds to actually 30 seconds to try to any calculator what happens if you do it lots and lots and lots of times.

31:58
Does it head off to infinity, wait wait wait, some of us, we've had some infinities you had some zeros, give you another 10 seconds.

32:11
Okay typing answers now off some of you typed in too early, are lots of zeros are fabulous. So are we are we agreed that it's getting really, really small. Is it ever really zero. Well, if I go to a 1 million the 1,000,000th iteration, it's going to be absolutely miniscule but still not quite zero. If I go to the 1,000,000,000,000th one, it's going to still be really small but not quite zero. But if I go what's called, in the limit if I go on forever. Then I do get zero, that's it's getting smaller and smaller, so I'll save in the limit, the area is zero, so I have a shape when I keep on going forever. You can draw it, and there'll be zero area. Okay, so I've got a shape with zero area. And that's that's mathematically what we'll say it's the infinity of Swan is what you get in the limit when you do it enough times, it's a bit like the strange idea that 0.9 recurring is actually one. If you stop at any point is not quite one, but if you go on forever. We can't distinguish it from one, if you do one minus naught point nine recurring. It's 0.00000 so, in fact, that there is no point at which it's anything other than zero so we'll say that 0.9 recurring and one of the same thing. This is the same idea that we go on forever we get zero. Okay, so these areas here I can cope I've got shape which I've taken essentially taken everything else off and I've got areas here. And what's its perimeter. So let's see if I go back to the original shape the perimeter of this shape is going to be, since the area is one the perimeter is something going to involve square root of three, I don't know if too complicated can't works out, I'll just call it P, for simplicity. So, if this shape to start with has perimeter P. When I cut out that smaller triangle and again I'm going to ask you to wait before you type in an answer, please. I've now got this line out around here and I've got this line around here, and I've got this line around here, so if I count the whole perimeter of the blue shape that's both the outer bits and the in bits, what's the perimeter. So I'll give you 30 seconds think about that, what is the perimeter of this shape.

34:57
321, go. Oh, I see lots of 1.5 peas or three half peas. So when I said before, I prefer fractions to decimals because otherwise the decimal is going to get really really messy, so I'll go for three halves P. I know in the chat, you can't take that very nicely but that's I know is what you meant. Absolutely. So I think you'll find the next one, much easier now we're getting into a rhythm. If I go to the next step. What's the primitive that give you 20 seconds

36:02
321 Go way to nine quarters P, and nine quarters, just as, Oh yeah, you're all getting this lay Whoo, good job guys. Just as we wrote 339 sixteenths we wrote his three quarters squared. In the same way this is three halves squared. So no guesses this time because when I go one more stage again I'm going to multiply the from here to here, I multiply the perimeter of each of those triangles by another three halves. So this one has perimeter three halves, cubed, p. So now if I go on forever. What's going to happen to the difference of weight. So I'll give you 20 seconds to have a think about you might want to type three halves three halves squared three yards cubed into your calculator perhaps what happens to the perimeter. As I go on forever. Have a think about it 20 seconds

37:24

321 Go in increases, increases, well this is harder math Sarah I like math Sarah infinity infinite got lots undefined infinity infinity, it's just getting really big, really big, really really really big infinity, just grows without limit, every step you go gets 50% Bigger multiplies by 1.5 multiplied by three halves the numbers getting bigger and bigger and bigger. Heading off towards infinity. So what we'll do is we'll say the perimeter of the ultimate shape, the actual Sierpinski triangle which is what you get if you go on forever, has Area Zero perimeter infinity. Ooh, that's just seriously weird. I've got a shape that I mean, I could fit onto this screen, I it fits within this triangle, it's not a big shape, but it has infinite perimeter and zero area. That's just, just weird. But maybe it was only Sierpinski who could do something like this, maybe it's just a really weird phenomenon doesn't care elsewhere. Until, along came home, I was going to say that I forgot. You can do it in three dimensions as well if two dimensions is not enough. What happens if we try the same thing, three dimensions, what happens if we start with a three dimensional pyramid. So, a tetrahedron is its mathematical name some tetrahedron, chop out the middle tetrahedron. And then from each of the remaining tetrahedron was actually bit more than the tetrahedron, you're chopping up the middle class and you're left with four smaller tetrahedra each half the size. And then from each of those you chop out the middle part and you're left with four smaller tetrahedra again.

39:21

The world's largest to Pinsky pyramid, otherwise known as a TETRIX at the Cambridge Science Festival on the 15th of March 2014 It's a fractal. It's lots of large pyramids, made out of smaller ones. It's lots of large triangles, made out of smaller ones. Imagine it keeps getting smaller and smaller and smaller to infinity, go the other way, it gets bigger, to infinity to

39:57

say you can do it out of modeling balloons in three dimensions as well. And if we asked about this shape what's its volume. Well if we started with volume of one for the entire pyramid what we're left with. When you do it forever. You chop out all those smaller things you left it smaller and smaller, smaller pyramids and in the limits at the end, you're infinity as one would have volume zero, but the surface area would be infinite. Whoa, crazy, and we can do it more theoretically as well so not a mathematician by the name of Karl Menger, born in Austria and eventually moved to America. He came up with a more three dimensional one which we can analyze in the same way. He called it or somebody called it after him, a manga sponge started with a cube is our level zero version, or stage zero. Let's assume the side length of this cube is one that means its volume is one and surface areas six because six faces of the cube, each has area one so side length one volume one area six. What I'm going to do is I'm going to chop it in to lots of small cubes so I'm going to divide each edge into three. As you can see along here, and I'm going to cut out the middle cube, the middle square sorry for each face so got this face here and I'm going to chop up middle square and what I'm going to do is I'm going to take like a square drill, and I'm going to draw the whole way down from the top, the whole way through the cube all the way out of the bottom end. And then from this side I'm going to draw a cubic hole all the way from this and all the way through to the other side and similarly from this face, all the way to the other side. I started with side length one. What's the resulting volume of this shapes, I'll give you a minute to think about it, what's the volume of the resulting shape, please don't type in answers yet, give you a minute to think about it.

42:54

Not five seconds or so. Okay, so it was a bit trickier this one so I'll give you time to type some answers in now 321 Go. Oh, I think we have lots of 20 over 20 sevens Oh, but I like 20 over 27 I think we are very much in agreement. All lots of 20 over 20 sevens there. Yes, I'm, I think we've all got it. So, take, you've got 20 cubes left, each of which is a side length for the third. So volume 27 120 730-423-2527 Good job. Good job team like that. What about the surface area now with Be a bit careful with a surface area because if I take, take a pen and draw this. I got all of this surface area along here, but I also have this little bit here and I have this little bit here and I've got a bit under here and I've got bits around here that you can't see as well. So, give you a minute to figure out what's the surface area of this shape. We started with surface area of six, What's the surface area of this shape. Different

45:20

and get ready now 321 Go for it with your answers.

45:36

Oh, we've got lots of disagreements here, all this is all this is going to be fun. Lots of disagreement, we've got some fours, we've got some eights we've got 72, we've got some. We've got 70 or 54. We've got 54 I've got lots of 23 of the three. This is proven really hard, hasn't it. Okay, let's try and figure it. What's this error, the area of one of these small faces here side length was one. So the area is one night so each one of these is one knife. So let's count on this side I've got 12345678. And then I've got my four in here 789 10 The one under here 1112 So that's this face plus the whole is 12, and I've got six of those so I've got six times, 12 have I left out anything in the middle, perhaps, is that covered everything was there anything I've left out somewhere.

47:00

Let's see if I walk say around this. Does anything change if I walk around this sort of stuff, square here, I've counted this face. On this side, counted this face on this side, this one that's hidden gets counted through this hole, and that one's hidden gets counted through this hole, so I've counted everything around this little cube. And I've counted all the ones around that cube and that's really all I've got, isn't it, so. So yeah, six times 12 times one nine, which is six times 12 Nine, which is six times fourths thirds, which does equal eight. Now I started with a surface area of six, so it's going to be just as we saw with the Sierpinski triangle, it's going to be more useful to think of it is six times four thirds. For the next step. So what have now ended up with is forget that automatic is I've got here 20 cubes, each with side length a third, the volume of the shape is 20 over 27. The area of the shape. Surface area is six times four thirds which is equal to eight. That was level one, what happens when I go to level two. So I'm going to turn each cube, I'm going to do exactly the same thing to it I'm going to cut a hole through the top through the two sites. So I had. I went from one cube to 20 cubes when I went from level zero to level one. So now I'm gonna turn each cube into 20 even smaller cubes. Now each side length is a third of what it was before so 1/3 squared. So the whole volume is 20 squared over three. Now I've got myself is 20 squared cubes, each of which is this, oh I've completely confused myself. It was 20 over 27. Sorry, it's, it's, uh, I should stop talking and think, can you help me out. What should the area be sorry. What should the volume be. I've got 20 squared cubes side length of each is, and nine. So it's 20 over 920 squared over nine cube, this does not seem very nice stuff. I'm going to write because I'm confusing myself. Very easy to try and think things in your head but when you it's very easy to get muddled. So when doing that, very helpful to write things down, so I had 20 cubes in the original one. Level one I had 20 cubes, each of which was 1/3 cubes in volume so I had 20 over 27. I've now turned each of my 20 cubes into 20 smaller cubes so I've got 20 squared. And each of them has side length one over three squared 1/9 So the volume of that is one over three squared cubed. Now, I could rewrite that as 20 squared times. If I've got one over three squared or cubed. I get one over three to the power of you should be able to think what I'm about to write six, three to power of six, which is equal to 20 squared times one over three cubed squared so I've all I've done is I've swapped swapped the three in the two effectively which is 20 over 27 squared. Ah, now it makes sense to me, I don't know if it makes sense to you. Let's see why. When I started with volume one I went to level one. I ended up with 20/27 of the volume for level two. I should have been able to guess that it's 20/27 of 20/27

51:41

haha so the volume left is 20/27 of 2027, so 20/27 squared, and the surface area is going to be level one I multiplied it by four thirds. So level two. I'm going to have to multiplied by four thirds. Again, just as we saw in the previous examples, we have. So I've got 20 squared cubes, each side as 1/3 squared, the volume is 20/27 squared the area's six times four thirds squared. I am not going to attempt on PowerPoint to draw level three because it would take me. Just too many days to do. So you're going to have to imagine it, I take each one of the small cubes that's left and I drew out the hole from the middle. As I do this again and again, again, in the limit, I end up with what's called the manga sponge. What is its volume. So I'll give you 10 seconds think about what's going to happen to the volume and they're not ask you to type the answer in this should seem somewhat familiar 321 Go. We have lots of zeros there, I'm liking this. Okay, it's going to go down to zero so we've got sheet with take out so much. We've got a volume of Zero good job you all well done 10 seconds think what's going to happen to the surface area as you go to infinity, as you go again again go to the limit, what happens to the area, surface area 10 seconds

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321 Go. Oh, you were so ready for it this time. Well, Guy is to infinity. So, whoa you are doing well. This tells me that we've got a shape that has volume zero and surface area infinity. Another strange shape so it's not, it's not just say Pinsky finding one random example manga found another. Okay, where do we go with this. Well, turns out you can build a level three, one, not some computer because that would just be too painful. This using PowerPoints, drawing, but you can make it by hand. So up here is Alison Kiddle he used to work friend Rich down his Kristin Caldwell she works for the advanced math support program, and I was involved in this, I was one taking these photographs. We're building a level three Sponge Out of business cards. So we, and it was a project called mega, mega manga, run by Matt Parker if I'm not very much mistaken. And we're building, and we take six business cards and we fold them together to make a cube. And then we take eight of these cubes to make a ring as you can see Allison's got a ring here and then you. If you get 20 of them you make one of these things which is was our level one. And if you take 20 of these and put them together you get one of these things. And if you take 20 of these things together, you get a level three sponge. So this is it in the middle of construction, and it's takes a lot of time, so think how many would take, you've got, well this is a group who has

finished it, they're up in Manchester. There's another group from Finland who finished there's a group in China finished stairs, and you've got 20. In each level if I go back to the previous one. There are 20 cubes in this. There are 20 of these in one of these bigger cubes so 20 times 20 And two, when you go to the whole thing the one that these groups have made, you've got 20 times 20 times 2020 cube that's 8000 cubes, each of which has six business cards in it. That's 48,000 business cards we use to make it. Unfortunately Don't with me have a photograph of our finished one in Cambridge. I haven't been to Cambridge best farmland for a year and a half because Coronavirus but last time I was there, we did have a finished level three sponge sitting there in the entrance hall to the math department. So if one day you're lucky enough to come visit Cambridge, you can see one of these, for real 48,000 business cards are just hours upon hours upon hours of work to make it, but very good fun. And it turns out there are other things you can do this a sponge like a sponge might make you think of like a sponge cake. Some of you may have eaten a sponge cake in your life. Some friends of mine made a manga sponge cake.

57:05
That's

57:07
plain plain sponge cake and chocolate sponge cake. And I think it's gingerbreads gingerbread layers to hold the whole thing together. So you can see some gingerbread days, and it is delicious. So if you want to go home and impress your family and friends, you could make yourself a manga sponge cake. Thoughts. Now, didn't go down too well. I hope you enjoy that, feel free. I'm sure you can find a recipe for manga sponge cake online. So I wanted to come back now after the practical making sponges, to think about how we might describe these things, and coming back towards the idea of fractals. So, one mathematical thing you've learned about at GCSE is similar shapes. Here are two sectors of circles, and they are similar, similar meaning I can take one and I can multiply it by something then maybe move it around or rotate it. I mode. In this case I've multiplied it by a factor of a half. and I've ended up with a smaller similar shape. It may not be an enlargement, there may not be a specific centre of enlargement I could have perhaps rotated it or something. But it's half the size of the original one. And so those shapes we call similar, and that should be very familiar to you from GCSE maths. We can see some similarity going on in the Sierpinski triangle as well. So here what I've done is I've taken the Sierpinski triangle, and I've highlighted. One of the triangles within it. So this triangle in the infinite case where we've gone on to the limit when we've gone on forever and ever and ever. And this triangle in the bottom corner is actually just the same as the original thing we started with, it's just half the size. So cop we've got copies which are half the size of the original. And how many copies we have three copies. So we've got copies which are half size of original we've got three copies of itself. What about the manga sponge. Similar idea. This time, the little red cube that's highlighted, I should have maybe had dotted lines around the ones you can't see, but that cube there in the corner is a third of the size of the original one. And, and if you've gone on. You've got an infinitely far so you've kept repeating this process that is genuinely a copy of the, of the whole thing, and it's a third of the size of the original and the shape is made of 20 copies of itself. So we've got a shape that is similar to itself. And so a fractal has that property. So one of the typical properties of fractals in some fractals at least is they've got some sort of aspects self similarity where some part of it looks in something like the whole thing. So that's a one aspect of typical aspects fractals. Another is that you have detail at all scales. So when you

zoom into a fractal and you look just at this part, it still looks incredibly, incredibly detailed. When you look at when you zoom into this tiny little cube here. If you've gone on lots and lots of levels, and it would also look still very very detailed, it doesn't sort of become simpler and simpler as you look at smaller and smaller bits. So that's two typical aspects of fractals there's another one which is possibly even more significant. And for that we need to talk about dimensions. So I'm going to tell you something you already know you already know, really, really well, and you've known for years. This is a straight line or straight line segment and we call it one dimensional. This is a square, we call a square two dimensional. And this is a drawing a cube and we call a cube, three dimensional. And I haven't told you anything you don't know. On this slide,

1:01:40
but now I'm going to ask you a really really really hard question, which you probably don't know the answer to, which is what makes this line segment one dimensional what makes this square two dimensional what makes this cube three dimensional. If I just showed you, cube and said how many dimensions is, does it have, how would you know the answers three. So give you a minute to think about that what makes it one, two or three dimensional. So give me a minute to think then I'm going to ask you to type some ideas into the chat, say go for one minute think. Okay, so I'll give you three to one to want to type some answers in.

1:03:27
Oh we get in lots of ideas, all this is really nice. Thank you. Really nice. Something two directions and axes that's that's really cool. How many axes, it has. These are really nice ideas. Thank you everyone and

1:04:08
direction so I've got lots of things about axes, height, weight, height, width, depth through different angles, coordinates needs specified points on the shape or some really nice ideas. So something to do, like different ways you can view a shape or number of different axes, different axes, perpendicular to each other. Then I'm going to get. I'll probably know what I mean by perpendicular. Like, surely perpendicular, in some sense maybe presumes I already have axes. So, if I have a three dimensional coordinate system, and I can draw my square within just the xy plane, then I can say it's two dimensional because I've only needed to use like the x and the y. And then you go, Okay, I may be just about by that. What would happen if I drew like, I don't know, a line, a line segments and then another line segments so right angles to it with that then make it two dimensional like Tic stick to one dimensional line segments next to each other. I don't know it is, I mean, this is not an easy question right if I drew that shape would that be two dimensional, or is that one dimensional, if I if I took a piece of paper which is a two dimensional thing and I fold it so now sort of, I don't know if you can see my pictures as your wedding see my camera. If you take a flat piece of paper and then you fold it so does it suddenly go from being two dimensional to being three dimensional because I folded it and it now needs a third dimension to describe it.

1:05:59
Hmm,

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how questions right. I think these are, these are genuinely hard questions, so I like, I love your ideas are really nice, and I'm going to share with you a different way of thinking about dimensions. I will tell you up front, that there are lots and lots and lots of different attempts to define what we mean by dimension. So, it's something that mathematicians have struggled with over, now more than a century until maybe 19th century, everyone knew what dimensions are and what you've written is absolutely agreed, it's like number of coordinates you can use the all sorts of problems started coming in around the, around 1900 as people started trying to really pinpoint what we mean by dimension. And so there are lots of different people try to understand what we mean by dimension, and there are some crazy things out there. I'm going to skip that slide. Piano did some crazy stuff he found a line that was so, so bent that it filled the hole of a square, every single point, the line with this line just went through every single point of the square so completely through anyone's concept I mentioned that window. Now the magician for you. We do love magicians, they do some amazing stuff, but that's another story for another day. Here's what one person came up with with the idea of dimension. And we'll use this to try and understand how we can describe the dimension of the Sierpinski triangle in the manga sponge. So they have this really interesting idea, they said, Well, you know, if I take a one dimensional shape or line segment. I can chop it into three smaller line segments. So, each one is a third of the size of the original, and I get three copies. I take a line segment and get three copies each a third of the size of the original. And I'm sure your jaws are just dropping in shock at this, this deep astounding astonishing statement, right. Okay, maybe weren't that excited by that. Fair enough. Let's see what happens in two dimensions. We know the square is two dimensional, I'm not, I'm not telling you today the square is not two dimensional squares, absolutely two dimensional. If we chop a square into things which are 1/3 the size of the original, as we did on the manga sponge. We get nine of them get nine copies each a third of the size. And with the actual cube itself, if we start with three dimensional cube and chop it into things which are a third of the size of the original, we end up with 27 of them 27 copies each 1/3 of the size. I hope none of this is particularly surprising. Let's put those together though, how many copies do we have, which are a third of the size. When we had a one dimensional shape we had three copies. When we had our two dimensional shape we got nine copies. When we had our three dimensional shape. We got 27 copies. Now, does those numbers mean anything to you will make you think of anything. I get 10 seconds and then I'll invite you to type some thoughts into the chat

1:09:47

321 Go square numbers powers of three powers of three powers of three geometrically is powers of three all lots of you've spotted its powers of three indeed powers of three powers of three power three increased by a factor of three powers of three scale factor, or we'll come back to scale factor in just one second. So let's write miss powers of three because that's what most of you said, I guess that's what most of us say three is three to the power of 193 to power of 227 to three to power of 3123 Does that match something salutely It matches the dimension. Oops, so that power matches the dimension of the shape. Three is the 132 wedges the three come from eight power times being the time being the dimension ATAR of x being dimension, yes. So, three power of dimension one is the number of copies you get three to power of dimension two is the number of copies you get three to power dimension three is the number of copies you get. Why is the base, three in each of them where does that. The bottom number three come from that comes from each copy being a third of the size of the original. So I've got a third of the size of original raised to the power of one race power of two race power of three, if

I changed the number of copies instead of the copies being a third of the size of the original let's say that 1/10 of the size of the original, then I would get n to the power of one or n to the power of two n to the power of three copies. So if I had cubes, each of which was one emphasize I'd have n times n times n, smaller cubes. So I get n to the power of one, and power to n to the power of three, it's n to the power of the dimension, and I can make a little summary of this rule. If the shape has dimension d, you'll get N to the D copies each one nth, the size of the original. And that works for any shape where you can cut up the shape into copies of itself into identically sized copies itself so works with a rectangle or a cuboid or straight line, it doesn't work with a circle. So this is a very limited idea of dimension, but it's, it's a good starting point. It's the first time, where we don't have to make any mention of sort of external axes or anything where we don't have to make talk about a graph or or some coordinate system is just about the shape itself and how the shape itself behaves. So, what we might call intrinsic property of the shape. So that's dimension d has. If you're the shape has dimension d then you'll have N to the D copies each 1/10 the size of the original. Let's try that with our sippin ski triangle. If we go back and I've kept this comment at the bottom this box at the bottom. We said we had this red one, we've got copies which are half the size of the original. This red one is half of the size of the original. And we've got said that the number of copies is two to the power of c, because if I put n is two in this I get two's power of d. And how many copies did I have, I had three copies. I'm sorry, these come out to the wrong board. There are three copies. And I said, Number of Copies according to my rule is n to the power of the dimension here and is two. So two to the power of dimension, that is, I need three to be equal to two to the power of d. Now, eyes, I don't know how many of you got it but I sent around a little sort of activity to try before the session. I don't know how many of you have done it but I'll give you a minute see can you come up with a number D, which solves the equations, three is equal to two to the power of d.

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Getting one decimal place is absolutely fine. Okay 321 Go for it. Whoa, got lots of numbers 1.6 1.6 1.6 1.58 Very good, very good. That is indeed, It's about 1.6 1.58. And I saw some a couple of people put in something about thought ice caught something. Logged somebody had a log in their answer. So one of the questions was, could you find a button on the calculator would do it it turns out that for this question de is about 1.6 1.58. And there is a button on the calculator that calculates it for you. It's called the log button, and it works solves the equation three equals two's power of d is log base two of three. Or, and some people have got other ways of writing it, but absolutely, I know logs are not in GCSE so I'm not expecting you to know that, but we can calculate an approximation on calculator and all the log button essentially does is calculates an approximation very very quickly for you. Okay, we can. And let me just make one more observation, we said that this shape had zero area, which sort of makes sense because it's dimension is less than two. If it had dimension two then it would have an area you'd assume, like, some sort of normal area that is that is less than two it's less contents than, then a square, but it's also bigger than the line The line has dimension one, this has got dimension one and a half or there abouts. It's bigger than the line. So that's why you have infinite, infinite length because it's just way even bigger than a line. So you might say, well, actually, I could go a bit further even and say, Well, when you talk about lines you measured their length when you talk about squares or circles or triangles whatever you measure that area. When you talk about cubes or pyramids or whatever you talk about their volume. So, length is like it's one dimensional, how much one dimensional stuff it has in it the posh name is content, how much one dimensional content, it has. And when you talk about area

you're asking how much two dimensional contents that have when you're asking about volume you're asking is three dimensional content. So when you've got this shape here and you ask how many how much one dimensional contents that have its length was dimensions too big for one dimension content, it has infinite one dimensional content. When you're about these two dimensional content, what its dimension smaller than two so it has 02 dimensional content zero area. Could I ask what it's 1.585 dimensional content is. Turns out it's got finite 1.585 bla bla dimensional content. I'm not going to tell you have to calculate that 1.585 dimensional content, and that would take us too far, a bit too much for today, but you can talk about it. If you look at dimension, anything bigger than 1.585 its content is zero because dimensions small now if you look at anything with dimension less than 1.5 85 say it's 1.5 dimensional content that's infinite because it's bigger dimension, so you can actually talk about something between length and area, if like 1.585 dimensional content, mind blowing. This is all invented in the 20th century. And we can do exactly the same with the 3d world, the thing that lives in three dimensional space, the manga sponge, the copies are 1/3 the size of the original and the number of copies, again by the same rule is going to be because it's 1/3, the number of copies is three to the D. We had 20 copies. And so we don't need 20 is equal to three to the D. And you can go and work it out.

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You can go and work it out on your calculator, and again, the answer is going to be somewhere between two because three squared is nine and three, three cubed is 27 so it's somewhere between two and three, log base three of 20, or you can find an approximation doing charm and improvement, and you'll find it's about three point is about 2.7. So again, this thing has finite 2.727 dimensional content, but infinite area, because its dimensions bigger than two and finite, and zero volume because its dimension is less than three. Crazy. So, we can give the third key thing to what a fractal is we said if exhibits some sort of self similarity has detailed or scales. And the third one is it has non into the integer dimension. So it's dimension is not a whole number. And we call, and that's where it gets the same fractal from fractals short for fractional dimension fractional here not meaning, a literal fraction but rather something that's not a whole number. I also mentioned very briefly at the beginning that none of these are completely deterministic, these are not precise definitions of fractal use there are fractals which are not self similar. They're, they're all fractals all have detailed all scales, but there are fractals which have dimension one, but still have fractured fractal behavior, even though they have dimension one, but that's sort of some of the properties of the fractal. I also disturbed you possibly by telling you that that dimensions were difficult to define and we struggled to come up with, you came up some really nice ideas of what dimensions are, but it's trying to pin them down like the dimension of the surface of a sphere is to even though you need three axes to draw a spheres, but that surface you think is like two dimensional thing because it's like flat. Unfortunately, that doesn't work with our this definition of dimension down here about copies, because you can't cut a sphere into small copies of itself. So, that Dummett definition of dimension that we came up with here about is called the self similarity dimension, it's very limited it only works on shapes which really genuinely are similar to themselves. So, you need to come up with a better definition of dimension that works, and tells you this dimension of the surface of a sphere as to the dimension of this is 2.7 or whatever and this is 1.6, and a mathematician, early in the 20th century came up with a way of doing it. German managed by named Felix housetops. And, and he managed to come up with a definition of dimension that works for any set. Fantastic definition, I'm not going to explain it today, but you can look it up, it's quite technical but it works and it finds damage of any set, it's got some really nice properties, but in the last 3040 years people have come up

with all sorts of different definitions of dimension that work better in some contexts or work better in other contexts. There's still no universal agreement as to what the definite what dimension is and there's depending on the area of Matthew working people will do different things. And

1:23:05
I'm looking at time if

1:23:11
you BS whether I'm going to have time to do that, I had another example of some mathematical show do one more mathematical one I'll show you some pictures. Okay, quick photo, I could do one more mathematical example show you some, some more real world examples. And so, math, or real world. So you want to type into the chat and I'll go for this or whatever seems to be the majority, go for it.

1:23:38
Oh, you're making my life too hard, it's about a 5050 is about 5050 Okay let me then, go, go and show you. It's about really installed out of 5050 Okay, let me go and show you one real world example and then give you a taster of how we can extend what we've done, tiny bit more. So how long is the south coast of England. So this was a chap by the name of Ben one Mandelbrot who came up with this observation, he said. But I'll tell you what I did with it so I took a picture of England. This is from Google Maps I did this a few years ago, and I drew a line along the south coast that I hit here's my line, okay I went around the Isle of Wight I didn't quite follow the Solent but I went around the Isle of Wight and I drew around Sufism, took great care of this. This took about 10 minutes to do it. It looks much easier than it is with like real, real care to draw the south cusping the doorway down to Penzance. And I asked Google Maps, how long is the line I've drawn, and the answer is the south coast of England is 711 kilometres, which is quite a long way if you want to walk it. And, anyway, so I then just said, You know what, let's just check I've drawn it right if I take sort of this area down here sort of going down Devon into como fight How well am I drawing let's just zoom in and check you may be getting used to this idea of zooming in, let's zoom in and check. And I realized that actually my really carefully drawn line is just not actually very good look over here, I've drawn a line right through the sea, and over here it's cutting across the land. So, actually this is not a very good essence it's really 311 kilometers. But then when I zoomed in to this little place over here I have no idea where this is, I realized the line I've drawn is just terrible. Just terrible, because look how contorted and detailed the coast is. And it turns out, this is a bit like the trees that we started with right at the beginning, that a tree is got lots and lots of detail and the closer you look at the tree the branches brighten break into twigs and then twigs break into little smaller twigs and so on, and a tree so detailed and the more closely you look at the more detail you see, likewise this coast as we zoom in we see more and more detail. So it may be that a better way of describing coast is not to say that it's a line and has a length but rather to say it's sort of fractal like sort of perfect fractal but it's quite a decent approximation to fractal. As we zoom in we see more and more detail at more scale, more scales. So if we try to measure the length that length doesn't actually make sense, because just as we saw with the Sierpinski triangle, the length is probably infinite, because it depends on which scale you measure it at, it's not perfectly infinite because it's not perfect frankly because we're dealing in the real world rather than in the mathematical world, but we can zoom in and get some idea of we can work out an approximation, what's the fractal dimension of the coast and Mandelbrot has in the in the 80s tried to calculate what's an approximation of the fractal dimension of

various different coasts out of the country and got numbers between 01 point one and 1.7 1.8. So, some would like Norway's very high fractal dimensions lots of fields that England is much lower fractal dimension, and then give some sense of House of bendy and House of detailed contorted are the different coastlines of the world. So that's a real world example. One. One more mathematical example is we go back to this triangle we said, there are three copies of the big triangle, sorry, three copies, smaller copies each size a half makes up a bigger triangle. And so we said, so three is equal to two to the power of d and that's what we did just few minutes ago.

1:27:55
And

1:27:58
we likewise said. For this we divided them we said we could chop it into three 927 small parts, but actually another way of thinking about it is if we scale the original shape to a third of its original size. Then, the length is of the reduced one is a third power of one, the area of the, that one is a third squared, the volume of the cube, small cube is the third cube, this is your length, area, volume scale factors that you learned at higher tier GCSE. Okay, so maybe we can sort of turn what we did before on the Sierpinski triangle upside down and think in terms of this way up instead of the other way up. If you've got dimension d. Then when you scale the content by a factor of a third. So when you scale the shape by a factor of a third, you scale the length or the area of the volume by a factor of a third, to the power of dimension. So, when we scale to a third of the size, the content is scaled to a third, to the power of dimension. We could replace the third by S or scale factor. So with scale factor of s, the content is scaled by s to the power of d.

1:29:29
So if we go back to this triangle now so Pinsky triangle and this is everything up here is what we had before. So, copies of half, three is two to the power of d and what I just put on this last slide, was when you scale by factor of s, the content is scaled by s to the power of d. So here, what we've done is we've scaled. I'll come back to this line in a sec. we've scaled by a factor of a half. And that gives something which is a third of the size of the original assist when I scale by half, I get a third of the original. So I'm going to want my third. Oops, no, no, let me let me use a pen instead I would have written this off, I want to say. So, I want my third, to be equal to half to the power of the dimension. Because when I've scaled it by a factor of half. I want the content to be a third of the content of the original because three copies that make the original. So I want the third to be a half to the D should be some two. If I take this equation and I take its reciprocal, I get a third and I get one over two to the D which is same as a half to D. That's exactly the same equation. So, both of these, give me the same idea. And that tells me that this D, I get from this equation in exactly the same way, but I can do something else and say, instead of thinking about it in that way. I can say, Well, my whole shape is made of three copies of this. I've got this copy here, which is, has one half to power of the content. I've got the copy at the top, which has helped out D content, and I have a copy to the right, bottom right, which is again, half the power of d. So I've turned this equation sort of upside down, and I've got half to d plus half the d plus half the D equals one. I'm pro probably lost some of you there, don't worry if I've lost you. And that was fine because each shape was this a half of the original half to the D plus half to the plus half to D, but maybe I could make a shape where we have different scale factors for different copies. So here, I've

taken a square and I've just put some smaller squares inside it. So here's a square which is two thirds of the size the original 1/3 of the size original a sixth of the size original a third of the size the original. What I've done now is I take this nice sort of magenta square. And I make copies of it. According to that picture on the left. So I've got a two thirds the original 1/3 The original one, six, the original third of the original. And then I repeat, if I take this sort of slightly strange shape and I do exactly the same. The dotted line around it by the way is just to show you where the original square was. So I've taken this and I've taken two thirds of it, plus 1/3 of it plus 1/3 of it plus 1/6 of it and I get this some other weird shape and I can do it again, I can keep on going, like it's a weirder and weirder and weirder shape you've got the site's like staircase structure along here and then you've got these little dangly things. This might look like a bit of a strange mathematical weird tree but you can imagine that from something like this you might end up with something looks a bit more tree blank. And then I might ask well what's its dimension. So, this shape that I ended up with in the last corner is made up of a copy of itself that's two thirds the size, plus a copy of itself that's 1/3 size plus a copy itself that once one sector size plus a copy of itself 1/3 The size.

1:34:07

So I've got this thing has content, two thirds, to the power of d. This has content 1/3 The power of d. This has content 1/3 power of d, this has content 1/6 to the power of d. And when I add those four parts together I get back to what I started with. So two thirds to d plus 1/3 to d plus 1/3 to d plus 162 D has to equal one, there is no nice button on your calculator that will solve this for you, I'm afraid. But you can go work out an approximation and it's going to be somewhere between one and two, it's bigger than a nine, but it's smaller than us than a square, so some number between one and two will give you the dimension of this of this fractal. And I will finish with a theorem which says that this shape is indeed made of similar. So if your shape as we've done here in this shape is made of similar non overlapping copies of itself is squares and two thirds 1/3 cylinder non overlapping and they've got scale factors, two thirds, 1/3 1/3 and 1/6. Then the dimension d of the shape is given by that equation, which is just a general version of what we wrote up here. It takes some effort to prove this, and you also need to use a definition of dimension which allows us to talk about dimension this shapes that we've used Hausdorff dimension to do it, but you can show that this equation does give you a correctly, the dimension of this shape. So there's both a real world example of coastlines and a mathematical example of slightly more complicated self similar shapes which are beginning to look a bit more tree like the one example I didn't get to talk about one big example I didn't get to talk about, I did put it in my slides as a. If we going super super super fast and I'm running at the speed of light, I might be able to do it with you. I just didn't think we would was the most famous fractal of all is the Mandelbrot set. This is a picture of Mandelbrot, then we'll be Mandelbrot sitting in front of Cambridge in front of the Isaac Newton institutes in Cambridge, part of, part of the central Mathematical Sciences. In the early 2000s, and this is a picture of his most beautiful fractal the Mandelbrot set. And if you want to find out more about the Mandelbrot set. I believe that Caitlin is going to send out a sheet of problems and questions of investigations that I, I've put together for you, some of which are quite hard, so please don't be put off that there are some which you might enjoy some which are harder some which are easier, but on that sheet there is a link to wonderful, wonderful workshop that Charlie guild sale from enrich together with Claire, who's been sitting with us all this time and answering, lots of questions in the chat. Thank you so much Claire, and did at the Cambridge Science Festival this year it's a, it was a webinar so you can watch it online, it's on YouTube, and they explore this Mandelbrot set and show you how it's made. Well well worth watching.

I will finish with a mathematical joke. There's a question Benwell Mandelbrot his middle initials B and there's a question is what is the B and baignoire B Mandelbrot stand for. And the answer is Benwell B Mandelbrot and the B and mend Dunmore B Mandelbrot stands for. Then while the man with the hat joke, I am going to thank you and invite you if you want say I will stay on for a few more minutes so I can ask, answer any questions either you want towards myself or you want to ask Claire, and otherwise I wish you a lovely afternoon thank you so much for joining us.

1:38:13
Okay so Science Festival when working on Mandelbrot that's the size putting YouTube link into the chat. The idea of perimeter surface area becoming infinite with fractals does tie in with Banach Tarski paradox one yes so sick, we take the infinite and so could be multiple means to same so so Banach Tarski paradox is connected to a thing called the axiom of choice where you can take this sphere and you can rearrange it into five identical shapes which you can riff, each of which has volume, same volume as the original sphere. And that's a whole problem to do with what we mean precisely by measuring volume, and some really really subtle questions in. In the mathematics of met what's called measure theory, and we generally when we do measure theory when we try to understand what we mean by volume, and this is just ordinary three dimensional volume. We try and avoid those really, really difficult paradoxes by limiting what we'll be willing to consider the volume of. So, it's related in the sense of measure theory does come up when you're trying to understand fractals, you have to be able to understand how to measure things so the two fields are very closely linked, but not precisely not precisely balanced Tarski paradox, really great question. Thank you. And thank you thank you lots of Thank you Thank you. Pleasure. Thank you. Pleasure, pleasure, pleasure to have you.

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There's another question about just the Mandelbrot equation relates to the theory of relativity. Oh, my answer was not that I know of, but

1:39:53
I will say, that doesn't mean it doesn't. There's so much in the Mandelbrot set that's really, really not understood. A there's a recent relatively recent paper last, maybe 2001 I think to be observed earlier. No, I think it was observed earlier but only proven relatively recently, which is you can calculate pi by doing iterations, around the Mandelbrot set, and finding how quickly you end up going off to infinity is related to the value of pi. So if you start very very soon so if you look at the Mandelbrot video that is without iterations and doing stuff again and again and again, if you start very very close to the Mandelbrot set. And you iterate it takes certain number of steps to get out to infinity. And there's a number of video about this, so certainly related to pi but theory of relativity. I don't think so that's more theory of geometry of surfaces than than Mandelbrot set, which is a question of iterations. So, I'm not aware of a relationship. Thank you for the insightful lesson. I do enjoy. Thank you for coming. Can you ever have a truly one dimensional line, yes math lesson magically you can have a truly one dimensional line, but anything we draw in the real world we're working with atoms and molecules and pieces of paper which are all living in three dimensions. So, in the physical world you can only deal with three dimensional things, or four dimensional if you want to think about relativity in space time, but in the mathematical world, you can deal with, absolutely you can deal with one dimensional things, and we do all the time. In the real life example you gave, kind of like decimals, what's the number answer. Word

number answer depends on how many decimal points you're round to instead of how much you are zooming in. So the real life example I was wondering if you're thinking about coastlines. So the coastlines. Yes, it's an approximation, it's not a it's not a perfect fractal it certainly does depend on how much you're zooming in, but when you talk about the dimension of the fractal what you're asking is a question like, if I measure using say a one kilometre long ruler. How long is the line, if I now measure using a 100 meter ruler 10 times smaller, how long does the coastline now become if I measure using a 10 meter ruler, how much longer the coastline become information using one meter ruler, how much longer coastline become. And my question is not, then becomes not how long is it at any particular rule at length. But how does the length change as I go from one kilometre ruler to a 10 100 meter ruler down to one meter ruler, and how much, how quickly it gets bigger, is related to dimension if the dimensions two then, when I take the rule of 10 times longer I'd expect to thanks 100 times bigger. If the dimension is one I'd expect the length also maybe 10 times bigger, I can't remember. By taking that dimension is one, and I expect the size of room to make very little difference to the length of the case. And if it's like one point, then I expect is a bit small, it would increase. Maybe like 1.2 times seven point to make it 10 times. I can't remember the exact numbers off top of my head but it's to do with it. That's the rough idea. Great question. And, or, thank you thank you too well. And can you send the PowerPoint with complex numbers, and the PowerPoint is pretty huge so I could put it somewhere where it can be downloaded and then ask the office to, to share that, yes. But no, I am thinking about it, I think you do better actually to listen to on second thoughts to listen to Claire, the reason being that if you just look at my PowerPoint without the explanations that go with it. I'm not sure it will make that much sense. So, I think, on second thoughts, Better will be better to watch case video. This rates of Penrose tiles and game, life, and I'm sure there are relations, I'm not sure what they are Penrose has a fascinating because if you know about Penrose tiles, you know, you get nonperiodic tilings, but do they have any sort of large scale structures that you can then think of in a fractal way, I don't know, it's a really good question. Not a clue. Not a clue. 14 is damaged Dimension Zero yes a point. Yes, thank you for. Absolutely. Thank you. Pleasure, pleasure, a fractal is being used in medicine. That's a really good question. I don't know, I'm not aware.

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I think they can be used to do things like model lung lungs.

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Oh yes, yes, obviously, though. Yes, very, very good. Yes they are, they are lungs of Thank you cleared lungs, it's very good, very fractal like structures. So yes, I hadn't really thought of that one. Yeah. And probably branching processes in looking at the blood system, the veins and arteries and capillaries, that's sort of very fractal like, it breaks from large ones to smaller and smaller and smaller ones.

1:45:31
At once the question about chaos theory factors are certainly the Mandelbrot set is is very much linked to chaos theory. And with that is very, very slight changes in initial positions, cause very very different behavior. So you can move a extremely tiny amount, and your iterations do something completely different. And in the YouTube video we do talk a little bit about that.

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Absolutely, and that they're, they're intimately connected. There's another really interesting connection which is one of the first examples of chaos found was quite a thing called the logistic map where you iteration. I could probably type this into the chat,

1:46:23
I think we have a YouTube video on that

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video. Also I wait for clouds. There's a thing called the logistic regret the liquid thick mat way, it's a very very simple map you start with a number, x, and you turn it into K times X. Oh no, I shouldn't use x because often times sine t maps to some constant times t times one minus t. So it's just a quadratic and K is a number between one and four, I mean, you use less than one, it's really boring. Case number between one and four, and the exhibits really really interesting behavior its effects K and you just keep going, you've put a number of T and T into k times t times t minus, times one minus T to get new value, and then you do it again and you do it again you do it again for some values of k, it behaves very very very sedately for other values of k it behaves chaotically. And there's a really close link between the behavior of that and the Mandelbrot set, which is crazy. But the Mandelbrot set is fun and SOS is chaos. Chaos is amazing, very important to dynamical systems very important to understanding the weather very important in understanding all sorts of phenomena in the world. And even though things are chaotic, you can still make statements about them in general things so fabulous area of math, very closely connected.

1:47:54
I've put in a link to the YouTube webinar that I did with Vicki Neil from Oxford. On the logistic map. What I can't find, I can't find the enriched page on it at the moment. Type the name logistic mapping at least his maths makes a difference with what we.

1:48:15
Okay so, Adrian is asking a very challenging question since this is one of the areas of math that is impossible to prove shows how this question mapping complete consistent side was proven wrong. Am I right, I'm not sure what you mean by, it's impossible to prove. So, There are many many proofs about fractals. There are many open questions about fractals as well, that doesn't mean it's impossible to prove just we haven't been clever enough to prove them yet. So one question that was a really, really tough nut was, you look at, if I go back onto the slide if I go back to previous slide, if you go back to the picture Mandelbrot sets the Mandelbrot set is the black part of it, which is, and the black part is not fractal the back part is just a normal area has area, positive area, it's two dimensional thing. But the boundary between the Mandelbrot set, and the not Mandelbrot set is like this circle, bit, plus the bulbs, plus the tiny bulbs on the bulbs plus all these filaments plus all of the boundary of the front of what set is certainly a fractal and there's, I didn't say but you can go onto YouTube and look for and sounds like Mandelbrot. If you search for Mandelbrot and zoom on YouTube, there you go see some people have done these amazing videos where they've zoomed into the boundary of the nanobots, they seemed in and they've zoomed in and zoomed in. They've simulated like a power factor of 10 to power 34 or something like that, and takes 10 minutes to gradually zoom in, it's quite psychedelic, and just see how

much detail that is in that. So there was a question about 20 years, which is the dimension of the boundary of the Mandelbrot set. Is it, it's certainly bigger than one dimensional, there's no question about that, because it's, it's definitely a fractal. And there was quite a long time that people believe that the boundary is actually two dimensional. And it took about 20 years and still some was able to prove it was a major result they managed to prove that the boundary of the Mandelbrot set, is actually a two dimensional set itself it has dimension to what we don't know is whether it's Area Zero or not, that is still an open question doesn't mean that it's impossible to prove it's just we haven't been able to prove it yet much maths, we haven't been able to prove them many many many open questions. So the question of maths being completely consistent and decidable is, is a very different question. Kurt girdle already proved that maths is not decidable, or more precisely, there are maths, his his incompleteness theorems show that you in any sufficiently strong formal system, there exists statements which either undecidable within the system, or you can get to think would only go in consistency, or omega and completeness, so it's a very, very detailed technical results. More about that. There's a lovely book by it's quite old now but it's really really good read called girdle, Escher, Bach by Douglas Hofstadter, if I can get my cursor. Good you'll, and I can't count the, The, the amount over here I'm sorry, todo Escher, Bach an eternal golden braid. I think that's how he spelled his name but I'm not certain. Great.

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What is actually meant by mathematical proof. Oh my goodness, I'm in the middle of writing, or coming towards the end of writing a book about that, and I'm just talking about how to prove things nice or skirt around the question of what is mathematical proof.

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What is mathematical proof.

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In the simplest answer to that is, it's something that convinces other mathematicians that the statements you've made is correct. So, you make a statement. The sum of two odd numbers is an even number. And then you do something, to convince your mathematically literate listeners that that is in fact the case. And if they accept your justification, then your justification is considered a proof. That's a social construct, and would every proof convince every mathematician. Well no, So you may have a proof that convinces some mathematicians but not not others. This has led to some really interesting arguments over the years where somebody claims to have proven something and somebody else claims that the proof is not a valid proof there was a very famous problem called the Kepler conjecture which was about, about, oranges, or more precisely spheres, but the question is, you want to pack your, you want to pack a box of oranges, as efficiently as possible. Let's make it an infinite box of oranges so you don't have to worry about edge effects and like the size of the boxes. So I've got infinitely many oranges or Sears, all of the same size, what is the most efficient way of packing them in space or time. Use the greatest proportion of space possible three dimensional Orange is the product had been proven in two dimensions, hexagonal packing. And there'd been some, some interesting results in higher dimensions, but three dimensional oranges. The Kepler conjecture was that there's, I think it's hexagonal close packing or something is a very standard way of packing oranges is the most efficient way of doing it, and nobody could prove it, and in the 1990s or early 2000s I can't remember which somebody claimed to have a proof, but somebody else claimed that their proof was missing a

step one was incorrect. And this argument went on for years over backwards and forwards over, is this a valid proof, until somebody else came along gave a completely different proof which was accepted by the community. But the question of what makes a mathematical proof valid is a really really really hard one. In recent years, there's been an increasing interest in computer verification of proofs, so things will prefer systems which are designed to help iron out these cases. So you come up with a system of logic, and you say this is our logical system that we're allowed to work in these are the rules you're now to use in the logical system. And if you want to prove that the sum of two odd numbers is an even number, you have to justify every step within this very formal system of logic to point this computer can verify that every single step you've taken is a valid step in the argument, and if you get to the end of that and the computer says yes your argument is valid, then we'll accept that as a mathematical proof. So, whereas you could argue that sum of two odds is uneven because I can draw an odd number like using all multilink cubes and they come in pairs and there's one leftover at the end and then I got another set of muslin cubes in pairs and one leftover in the end and if I fit them together, I don't know if you can see my hands if you're fitting together it's not very good description. The two leftover ones match up and now I've got to just pairs, everything's paired up and so towards making even, maybe that's a good enough argument. So you've got whole spectrum of what's accepted as a mathematical proof, really, really good question, really hard question. And there's a whole area of, I guess philosophy called the philosophy of science or philosophy of mathematics, which asks questions like, What is a proof. And there are university courses on philosophy of science or history and philosophy of science, general advice is you do much better to study mathematics or science first until you really really understand subject from the inside and then if you're interested in the philosophy, to then to go over to philosophy because you can't really talk about the philosophy of subject until you have a really good in depth understanding of it. I hope that very long answer, or non answer has given you some sense of how good the question was,

1:57:27
any other questions. These are brilliant, brilliant questions and really enjoyed this. Thank you.

1:57:54
So, I suggest, in which case if there are no more questions I thank everyone who's still here. Thank you very much for joining today. I hope that's been really helpful, I wish you all, every success since you begin your sixth forms, and you begin your A Levels next year. I hope you have a very much gentler and more normal year, and being really nice meeting your virtually as it was. Thank you so

