1. <u>Submarine toy problem</u>

The sub will be on the point of sinking when

$$\rho_{sub} = 1000 \text{ kgm}^{-3} \quad \text{(neutral bouyancy).}$$

$m_{sub} = 14 \times 10^{-3}$ kg

$V_{sub} = 28 \text{ cm}^3 = 28 \times 10^{-6} \text{ m}^3$

At the start

$$\rho_{sub} = \frac{m}{V} = \frac{14 \times 10^{-3}}{28 \times 10^{-6}} = 500 \text{ kgm}^{-3}$$

Water is incompressible so all the squashing is done by the sub.

What is its volume when $\rho_{sub} = 1000 \text{ kgm}^{-3}$?

$$V_{sub} = \frac{m_{sub}}{\rho} = \frac{14 \times 10^{-3}}{1000} = 1.4 \times 10^{-5} \text{ m}^3$$

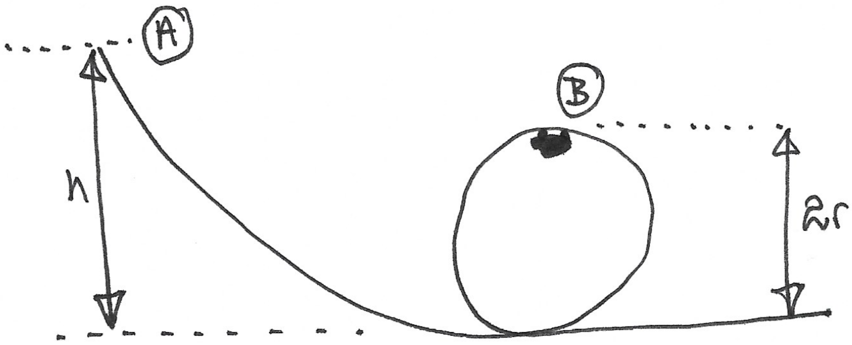
double density $\Rightarrow$ half volume

So the volume pushed down by the bung is $1.4 \times 10^{-5} \text{ m}^3$

$$\pi r^2 h = 1.4 \times 10^{-5}$$

$$h = \frac{1.4 \times 10^{-5}}{\pi (0.03)^2}$$

2. Loop the loop



Centripetal force:

$$F = \frac{mv^2}{r}$$

At the top of the loop, weight provides the centripetal force so

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{gr}$$

Energy is conserved so

GPE at A = GPE + KE at B

$$\cancel{m}gh = \frac{\cancel{m}v^2}{2} + \cancel{m}g \times 2r$$

$$\cancel{g}h = \frac{\cancel{g}r}{2} + 2\cancel{g}r$$

$$h = 2.5r$$

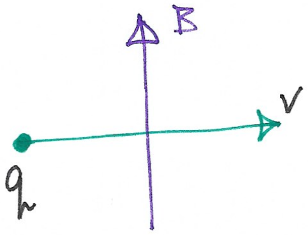
3. Magnetic force on a charge

$m = 10^{-29}$ kg

$q = 2 \times 10^{-19}$ C

$v = 10^6$ ms^{-1}

$B = 0.1 \times 10^{-3}$ T



$$F = Bqv = \frac{mv^2}{r}$$

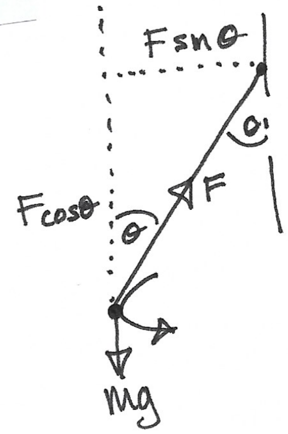
$$r = \frac{mv^2}{Bqv} = \frac{mv}{Bq}$$

$$r = \frac{10^{-29} \times 10^6}{0.1 \times 10^{-3} \times 2 \times 10^{-19}}$$

$$r = 0.5\text{m}$$

4. Conical pendulum

length = L
radius = r



$\rightarrow F \sin\theta = m\omega^2 r$

and $\sin\theta = \frac{O}{H} = \frac{r}{L}$

so $\frac{F\cancel{r}}{L} = m\omega^2 \cancel{r}$ ——— ①

$\uparrow \; F\cos\theta = mg$

$F = \frac{mg}{\cos\theta}$ ——— ②

From ① and ②

$L\cancel{m}\omega^2 = \frac{\cancel{m}g}{\cos\theta}$

$\omega^2 = \frac{g}{L\cos\theta}$

Meanwhile $\omega = \frac{2\pi}{T}$ and $T = \frac{2\pi}{\omega}$

so $\frac{1}{\omega} = \sqrt{\frac{L\cos\theta}{g}}$

$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

$$\boxed{\text{Centripetal force: } F = \frac{mv^2}{r} = m\omega^2 r}$$

5. Moon's orbit around the Earth

$r = 3.8 \times 10^8 \, m$
$T = 29$ days
$G = 6.7 \times 10^{-11} m^3 kg^{-1} s^{-2}$

$$F = \frac{G\cancel{m}M}{r^2} = \cancel{m}\omega^2 r$$

$$M = \frac{\omega^2 r^3}{G}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{29 \times 24 \times 3600}$$

$$\omega = 2.508 \times 10^{-6} \, rads^{-1}$$

$$m = \frac{2.508 \times 10^{-6} \times (3.8 \times 10^8)^3}{6.7 \times 10^{-11}}$$

$$M = 5.15 \times 10^{24} \, kg$$

Newton's law of gravitation:

force $F = G \frac{mM}{r^2}$ 2 masses

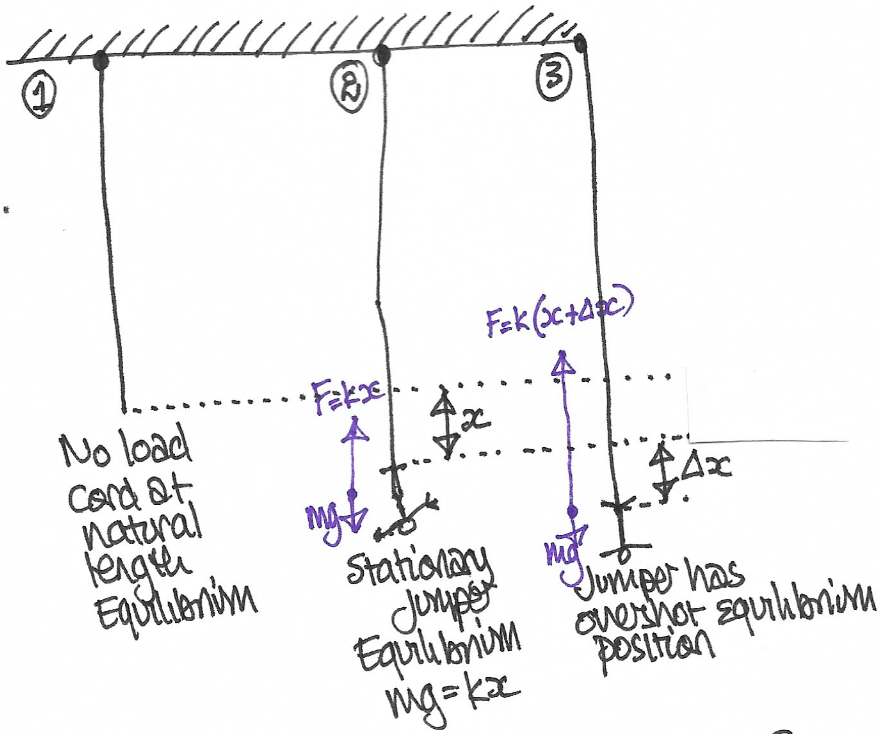
"Big G" AKA Newton's gravitational constant

distance between them

Centripetal force $= \frac{mv^2}{r} = m\omega^2 r$

6. Bunjee jumper bounce frequency

Consider 3 states of the bunjee cord & jumper:



In case ③, the elastic applies a restoring force

$k\Delta x = ma$ (from Newton II) ———— ①

Definition of simple harmonic motion (possibly a 2nd year topic)

$$\boxed{a = -\omega^2 x}$$ ———— ②

"A motion is simple and harmonic if the displacement is directly proportional to the acceleration and in the opposite direction"

From ① $a = \frac{k\Delta x}{m}$ and Δx is the same as x in ② - displacement from equilibrium, so

$$\frac{k\cancel{\Delta x}}{m} = -\omega^2 \cancel{\Delta x}$$

and $\omega = \sqrt{\frac{k}{m}}$ ← Minus sign disappears because its describing direction, not size

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$